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Genetic Analysis of *Analogique B*

Iannis Xenakis composed *Analogique B* for tape (1958-1959) concurrently with the instrumental work *Analogique A*. Both works utilize Markovian stochastic methods. However, the precision of the tape medium allowed Xenakis to more concretely explore his notion of complex sounds formed by grains of sonic entities, what is often called "granular synthesis" today.

A genetic analysis (an analysis by means of re-creating a work) of Xenakis' *Analogique B* created in real time and employing both explicit and implied procedures published in *Formalized Music* (1992) provides some advantages. The real-time algorithms provide unique instantiations with each realization; the power of contemporary computers allows for precise realizations of stochastic number generators without the estimations of hand-calculations; and, the immediacy of the realization enables listeners to hear different instantiations in sequence, allowing generalizations on Xenakis' procedures. However, the re-creation lacks a quality that is difficult to quantify. This paper discusses the methods and results of recreating *Analogique B* using real-time audio technology and noting the aesthetic implications arising from the result.

Granular Synthesis

Xenakis conjures a dramatic image of what is possible with grains of sound forming a complex sound. His analogy immediately evokes a process made up of iconic elements:

> A complex sound may be imagined as a multi-colored firework in which each point of light appears and instantaneously disappears against a black sky. But in this firework there would be such a quantity of points of light organized in such a way that their rapid and teeming succession would create forms and spirals, slowly unfolding, or conversely, brief explosions setting the whole sky aflame. A line of light would be created by a sufficiently large multitude of points appearing and disappearing instantaneously. (Xenakis 1992: 44)

This is, perhaps, one of the first published notions of what is generally called "granular synthesis" today. Though contemporary composers have developed granular synthesis to serve multiple aesthetic aims, Xenakis originally proposed to structure these grains using stochastic methods. As one of his earliest works, *Analogique B* was an example to prove the relevance of stochastic methods in composition and the efficacy of granular synthesis.

Micro-composition – Explicit Parameters

Xenakis developed compound probabilities in complex relationships in order to achieve his aesthetic aims. The processes underlying every aspect of composition begin with the most basic parameters of sound and accumulate to formal proportions. In "Markovian Stochastic Music – Theory" and "Markovian Stochastic Music – Applications" from *Formalized Music*, Xenakis described some parameters of *Analogique* explicitly. In other cases, Xenakis speculated what could be possible given computational capabilities. I will begin with the explicit parameters provided by Xenakis in his writings.

Frequency, Intensity, and Density Distributions

Xenakis identified three main variables of a complex sound as frequency (f), intensity (g), and density (d) of sonic grains. For each variable, Xenakis created two configurations of value ranges on a scale. He called each configuration "0" and "1" of the variable. Thus six variables representing distributions of values were created.

Xenakis provided the following figures (Xenakis 1992: 104-5):

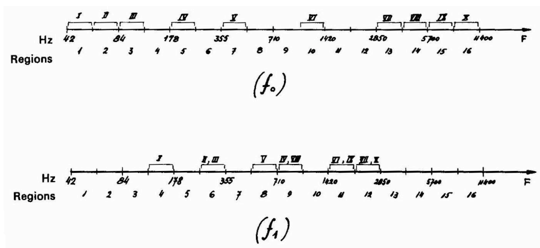

Figure 1: Frequency Regions

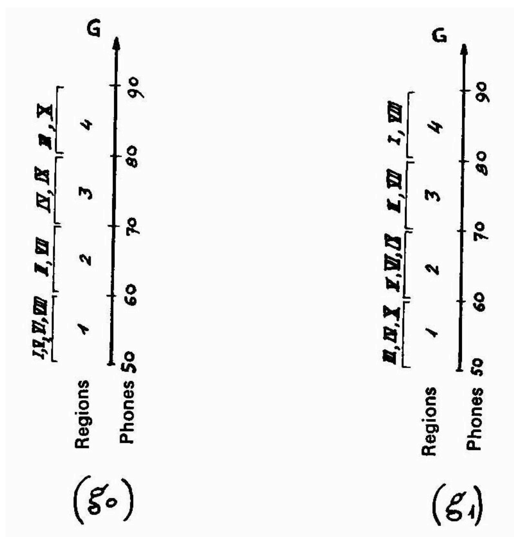

Figure 2: Intensity Regions

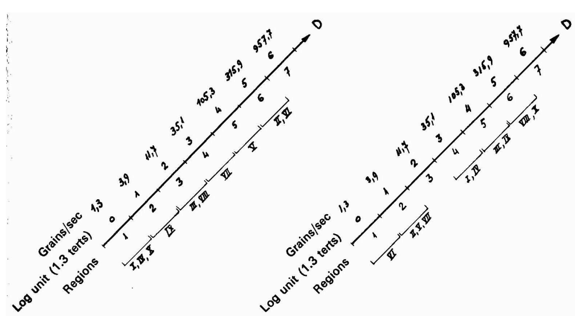

Figure 3: Density Regions

*Screens (*Trames*)*

There are eight possible combinations of three variables with two possible values. Each combination is a different configuration of ten clouds of grains in three-dimensional space. Though the configurations may be represented by three-dimensional graphs, those graphs tend to be difficulty to read. Xenakis combined these variables into two-dimensional grids, notating density as a number. Xenakis called these grids *trames*, translated as “screens.” In this representation, it is easier to read each screen as a slice of time in the life of a complex sound. It is also easier to see that Xenakis chose to define ten clouds of grains for each screen. The Roman numeral assignations make it easier to identify the individual dimensions on each screen.

$$f_0 g_0 d_0 = \text{Screen A}$$
$$f_0 g_0 d_1 = \text{Screen B}$$
$$f_0 g_1 d_0 = \text{Screen C}$$
$$f_0 g_1 d_1 = \text{Screen D}$$
$$f_1 g_0 d_0 = \text{Screen E}$$
$$f_1 g_0 d_1 = \text{Screen F}$$
$$f_1 g_1 d_0 = \text{Screen G}$$
$$f_1 g_1 d_1 = \text{Screen H}$$

Figure 4: Eight Possible Combinations of *f, g, d*

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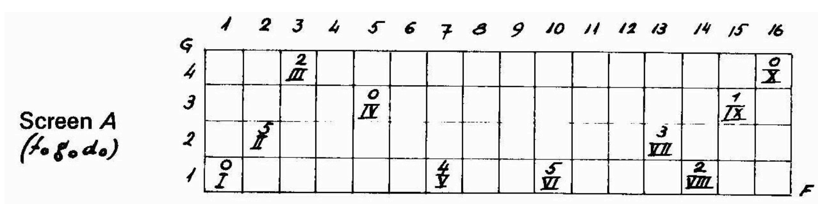

Figure 5: Screen A (Xenakis 1992: 106)

Matrix of Transitional Probabilities

Xenakis asserted that linking these screens together like pages of a book might describe a complex sound (Xenakis 1992: 69). The order by which the screens are linked is subject to stochastic methods. One such stochastic method is the Markov chain: a system in which the probabilities of each successive event are determined by the preceding events. In other words, a Markov chain specifies transitional probabilities, where the transition between the events is probabilistic. Xenakis depicted these probabilities in Matrices of Transitional Probabilities. In two cases, he created systems that are stochastic, or tend toward equilibrium states when unaffected by external impulses

An explanation of these Matrices of Transitional Probabilities (MTPs) is better done with a more general example. The first MTP, designated by the symbol ρ (rho), is given:

	X	Y
X	0.2	0.8
Y	0.8	0.2

Table 1: First MTP (ρ)

The second MTP, designated by σ (sigma), is:

	X	Y
X	0.85	0.4
Y	0.15	0.6

Table 2: Second MTP (σ) (Xenakis 1992: 82)

MTP (ρ) determines the probability of X or Y in a sample set. Take two urns filled with X and Y marbles. One urn has 20 X and 80 Y marbles, while the other urn has 80 X and 20 Y marbles. Data are collected in the following manner: draw a marble from one of the urns and return it, noting whether it is an X or Y marble. If one draws an X marble, draw from the urn with 20 X and 80 Y marbles. If one draws a Y marble, turn to the urn with 80 X and 20 Y marbles and draw from it. One possible sequence of draws may look like this:

1. Start with an X marble. Note it, and then draw from the first urn (indicated by the seed X marble). There is an 80% probability the next draw will result in a Y marble.
2. Assume the first draw results in a Y marble. Note it, and then draw from the second urn (indicated by the drawn Y marble). There is now an 80% probability that the next draw will result in an X marble.
3. The second draw unexpectedly results in a Y marble. Note it, and then draw again from the second urn. There is still an 80% probability that the next draw will result in an X marble.
4. The third draw results in an X marble. Note it, and then draw from the first urn.
5. The noted results are: X Y Y X

Assuming that this process is continued on, it is probabilistic that resulting draws will alternate between X and Y, with occasional repeats. Thus, a likely continuation of this process could be:

X Y Y X Y X Y Y X Y X Y X Y X X Y X Y Y Y

Figure 6: Possible draws using MTP (ρ)

In this simple case, each sample set is made up of one term (or one draw). However, it is possible to define the sample set with any number of terms. For example, define a sample set as ten terms. Therefore, ten draws are taken from an urn and noted. In this case, ten draws from the first urn would result in approximately two X marbles and 8 Y marbles based on the probability of X and Y.

If two X marbles and eight Y marbles are drawn in the first step, two marbles must be drawn from the first urn and eight from the second urn in the second step. This creates a combined probability. In the case of the two marbles drawn from the first urn, it is most likely that two Y marbles will be drawn. In the case of the eight marbles drawn from the second urn, the result would be approximately six X marbles and two Y marbles. The combined result is six X marbles and four Y marbles.

So, in the third step, six draws must come from the first urn and four draws must come from the second urn. The peculiar characteristic of this MTP is that it tends to an equilibrium state where the probability of drawing an X or Y marble is 50%.

Another way to express this process is that the ten samples determine the proportions of X and Y marbles in a new urn, from which ten samples are taken. These next ten samples then determine the proportions of X and Y marbles in a third urn, from which ten samples are taken. This process is repeated until equilibrium. Equilibrium is achieved when the proportions of the next urn are the same as the previous urn. Mathematically, this is expressed as:

$$
\begin{aligned}
&\text{Let } X = \text{the probability of drawing an X marble}\\
&\quad Y = \text{the probability of drawing a Y marble:}\\
&\quad X = 0.2\\
&\quad Y = 0.8
\end{aligned}
$$

Each subsequent urn is a combined probability using the values from the preceding ten samples. Therefore:

$$
\begin{aligned}
X' &= 0.2X + 0.8Y = 0.04 + 0.64 = 0.68\\
Y' &= 0.8X + 0.2Y = 0.16 + 0.16 = 0.32\\
X'' &= 0.2X' + 0.8Y' = 0.14 + 0.26 = 0.40\\
Y'' &= 0.8X' + 0.2Y' = 0.54 + 0.06 = 0.60\\
X''' &= 0.2X'' + 0.8Y'' = 0.08 + 0.48 = 0.56\\
Y''' &= 0.8X'' + 0.2Y'' = 0.32 + 0.12 = 0.44
\end{aligned}
$$

The urns begin with a higher proportion of X, then oscillate with each sample set. With each sample set, the oscillations reduce until, ultimately, there is an urn with equal proportions of X and Y marbles. This is mathematically proven by:

$$
\begin{aligned}
&X^{n+1} = 0.2X^{n} + 0.8Y^{n}\\
&Y^{n+1} = 0.8X^{n} + 0.2Y^{n}\\
&X^{n} + Y^{n} = 1\\
&\text{and, this system reaches equilibrium when}\\
&X^{n+1} = X^{n}\\
&\therefore\\
&X^{n+1} = 0.2X^{n} + 0.8(1 - X^{n}) = 0.8 - 0.6X^{n}\\
&X^{n} = 0.8 - 0.6X^{n}\\
&X^{n} = 0.5
\end{aligned}
$$

The system defined by MTP ρ reaches equilibrium with equal proportions of X and Y.

The second MTP (σ) behaves differently as it approaches and reaches equilibrium. The combined probability of each sample set is:

$$
\begin{aligned}
X' &= 0.85X + 0.4Y = 0.72 + 0.06 = 0.78 \\
Y' &= 0.15X + 0.6Y = 0.13 + 0.09 = 0.22 \\
X'' &= 0.85X' + 0.4Y' = 0.66 + 0.09 = 0.75 \\
Y'' &= 0.15X' + 0.6Y' = 0.12 + 0.13 = 0.25 \\
X''' &= 0.85X'' + 0.4Y'' = 0.64 + 0.10 = 0.74 \\
Y''' &= 0.15X'' + 0.6Y'' = 0.11 + 0.15 = 0.26
\end{aligned}
$$

Equilibrium is proven to exist by:

$$X^{n+1} = 0.85X^n + 0.4Y^n$$
$$Y^{n+1} = 0.15X^n + 0.6Y^n$$
$$X^n + Y^n = 1$$
and, this system reaches equilibrium when
$$X^{n+1} = X^n$$
$$\therefore$$
$$X^{n+1} = 0.85X^n + 0.4(1 - X^n) = 0.4 + 0.45X^n$$
$$X^n = 0.4 + 0.45X^n$$
$$X^n = 0.73$$

In this case, the system reaches equilibrium, but X marbles are more likely. These two systems display two very different characters. In MTP ρ, the urns oscillate between favoring X and Y marbles until they are equally probable. In MTP σ, X is always more probable, and the system tends toward its equilibrium state smoothly without oscillations.

Xenakis utilizes MTPs ρ and σ for each of the three dimensions of screens. For frequency there are two MTPs: MTPF α and β; for intensity, there are MTPG γ and ε; and for density, there are MTPD λ and μ. In order to link the three dimensions together probabilistically, the values of two dimensions determine which MTP (ρ or σ) is used for the third. Xenakis gave these coupling parameters in this configuration (Xenakis 1992: 83):

$$
\downarrow \begin{array}{cccccccccccc}
f_0 & f_1 & d_0 & d_1 & g_0 & g_1 & g_0 & g_1 & f_0 & f_1 & d_0 & d_1 \\
\lambda & \mu & \alpha & \beta & \lambda & \mu & \beta & \alpha & \gamma & \varepsilon & \gamma & \varepsilon
\end{array}
$$

Figure 7: Coupling Parameters for MTP

Rather than doing multiple calculations for each grain of sound when composing, it is far easier to calculate the combined probability of each screen transitioning to the other screens, although this means 64 calculations. For example, if the current screen is A ($f_0 g_0 d_0$), what is the probability that the next screen will be A again?

First, the MTPF must be determined by g_0 and d_0. The coupling parameters show that g_0 leads to MTPF β while d_0 leads to MTPF α. This signifies that the choice of α and β are equally probable. The probability that f_0 will remain as f_0 in α is 0.20 and in β is 0.85. The combined probability, therefore, is calculated as:

$$f_0' = 0.5(0.2) + 0.5(0.85) = 0.525$$

Similarly, MTPG is determined by f_0 and d_0. Both variables indicate that γ is to be used. The probability that g_0 remains g_0 is simply taken from γ: 0.2. Finally, MTPD is determined by f_0 and g_0. Both variables indicate that λ is to be used. The probability that d_0 will remain d_0 is 0.2, as well. The combined probability that $f_0 g_0 d_0$ (screen A) will remain $f_0 g_0 d_0$ is the product of all three probabilities: $0.525 \cdot 0.2 \cdot 0.2 = 0.021$.

When all 64 calculations are completed, they may be combined into one large MTPZ, the combined probabilities of MTPF, MTPG, and MTPD. Xenakis provided that calculated matrix (Xenakis 1992: 89):

MTPZ

↓	*A* ($f_0g_0d_0$)	*B* ($f_0g_0d_1$)	*C* ($f_0g_1d_0$)	*D* ($f_0g_1d_1$)	*E* ($f_1g_0d_0$)	*F* ($f_1g_0d_1$)	*G* ($f_1g_1d_0$)	*H* ($f_1g_1d_1$)
A($f_0g_0d_0$)	0.021	0.357	0.084	0.189	0.165	0.204	0.408	0.096
B($f_0g_0d_1$)	0.084	0.089	0.076	0.126	0.150	0.136	0.072	0.144
C($f_0g_1d_0$)	0.084	0.323	0.021	0.126	0.150	0.036	0.272	0.144
D($f_0g_1d_1$)	0.336	0.081	0.019	0.084	0.135	0.024	0.048	0.216
E($f_1g_0d_0$)	0.019	0.063	0.336	0.171	0.110	0.306	0.102	0.064
F($f_1g_0d_1$)	0.076	0.016	0.304	0.114	0.100	0.204	0.018	0.096
G($f_1g_1d_0$)	0.076	0.057	0.084	0.114	0.100	0.054	0.068	0.096
H($f_1g_1d_1$)	0.304	0.014	0.076	0.076	0.090	0.036	0.012	0.144

Figure 8: MTPZ - Combined probabilities for transitioning between all screens

The last remaining calculation in this design is proof that the combined MTPZ itself has an equilibrium state. Xenakis provided the proof that a region does, in fact, exist; and the screens have the following distribution in equilibrium (Xenakis 1992: 89-90):

Screens	Probability
A	0.17
B	0.13
C	0.13
D	0.11
E	0.14
F	0.12
G	0.10
H	0.10

Table 3: Distribution of screens in equilibrium

Micro-composition – Implicit Parameters

Though Xenakis clearly explained and detailed the construction of the screens and their transitional probabilities, he did not explicitly state how each grain was ultimately placed in the work. However, in his opening discussions of grains, he suggested certain favored stochastic distributions for these dimensions.

Xenakis suggested that the dimensions of frequency and intensity have Gaussian (Xenakis 1992: 55-6). As for temporal distribution, Xenakis implied that he used sinusoidal sounds whose durations are constant at about 0.04 seconds. They are placed in time as a function of their density utilizing the Poisson distribution (Xenakis 1992: 54).

The Gaussian distribution has a mean value and standard deviation within which the values have the highest probability of occurring. A Poisson distribution is similar to a Gaussian distribution; but it more accurately depicts the probability of rare events. These equations were used to generate the grains within each cloud. The mean values for frequency and intensity were determined by the centers of each frequency and intensity region. The mean value of density was determined by the mean number of events given by the density region divided by the time for each screen (about 0.5 seconds) (Xenakis 1992: 105).

Macro-composition

Definitions

The use of screens suggests a particular formal structure that utilizes their stochastic tendency toward an equilibrium state. It is necessary at this point to define and use Xenakis' terminology to discuss the formalization of music using MTPs.

Xenakis defined a *mechanism* as any transformation of an entity from one state to another (Xenakis 1992: 72). In *Analogique B*, each grain and its individual transformations are a mechanism. For example, in the original example of pulling a marble from an urn, the mechanism was an entity that could either be an X or Y marble. However, a mechanism may be made of multiple grains. Thus, the example where the sample set consisted of ten marbles was a complex mechanism consisting of internal mechanisms. The ambiguity in meaning can be misleading. In the case of *Analogique B*, it is most useful to think of the work as a whole as a mechanism, though it consists of internal mechanisms of frequency, intensity, and density.

A mechanism passes through many *states* (Xenakis 1992: 73). In the marble example, a state is "X" or "Y." There can be any number of states within a *stage*. A stage can be understood as a single urn with a set of probabilities. When one marble was drawn at a time, the stage had a *period* of one state. When ten marbles were drawn, the stage had a period of ten states (Xenakis 1992: 96).

Without some external perturbation, a mechanism will continue endlessly in its equilibrium state. In equilibrium, each successive stage has the same probabilities as the previous stage. Xenakis asserted that the stochastic quality (tending to a goal) of a mechanism can only be perceived when it is shown to consistently return to equilibrium after a perturbation.

The design by which a mechanism is perturbed (and the resulting stages toward equilibrium) is the *scheme* of a mechanism. Xenakis intended the definition of scheme as it means a plan or blueprint (Xenakis 1992: 81). In *Analogique*, a perturbation can be likened to starting with all X marbles in the urn example. In a perturbed stage, called P_A^0, the probability that screen A will remain screen A is 1. In other words, there is no probability; some external force is imposing a perturbation on the system. However, the next stage is one step closer toward equilibrium.

Since the previous stage consisted of only one possible result, the MTPZ of the next stage only has one column (Xenakis 1992: 93):

	A
A	0.021
B	0.084
C	0.084
D	0.336
E	0.019
F	0.076
G	0.076
H	0.304

Table 4: MTPZ of P'_A

If a composer were to choose to use all stages between perturbation and equilibrium, many MTPZs would need to be calculated. Xenakis simplified his task by utilizing only perturbations, the first stage after perturbations, and equilibrium (Xenakis 1992: 93). In all three cases, he only needed to use a single column of values. Once the equilibrium values were determined (see Table 3), he needed only to distribute screens according to those values, rather than cross-referencing every screen with the MTPZ and creating random numbers to determine each state.

Scheme of Analogique B

Analogique B shares the same scheme as its sister work, *Analogique A*. The scheme of *Analogique B* is:

$$E \rightarrow P_A^0 \rightarrow P'_A \rightarrow E \rightarrow P'_C \rightarrow P_C^0 \rightarrow P_B^0 \rightarrow P'_B \rightarrow E \rightarrow P'_A$$

Figure 9: Scheme of *Analogique B* (Xenakis 1992: 105)

Each letter indicates a stage of the mechanism. Each stage consists of 30 states. Each state is about 0.5 seconds long. Therefore, the period of each stage is about 15 seconds (Xenakis 1992: 105). Since the mechanism itself is complex (consisting of ten stages, which in turn have 30 states, which in turn are determined by a screen of probable grains), Xenakis breaks down the scheme into protocols. Therefore, the sequence of screens pulled from the urn is a protocol that determines the distributions of the clouds of grain. Xenakis provides an example of Protocol (E), the equilibrium state:

A D F F E C B D B C F E F A D G C H C C H B E D F E F F E C F E H B F F F B C H D B A B A D D B A D A D A H H B G A D G A H D A D G F B E B G A B E B B ...(Xenakis 1992: 97)

In contrast, the protocol for the perturbation P_A^0 would be:

A A A A A A A A A A A A ...

Theoretically, the micro-compositional parameters (screens), the micro-compositional inferences (distributions of grains on screens), and the macro-compositional information indicated by the scheme of the mechanism provide all the necessary information to re-create *Analogique B*.

Pd Patch

Pd is a graphical programming environment created by Miller Puckette. It functions similarly to Max/MSP software with an emphasis on signal processing. For *Analogique B*, I used Pd 0.37 for Mac OSX.

Protocols and Stages

The protocol defined by Xenakis is hard-coded into the patch. A counter, increasing every 0.5 seconds, triggers each of thirty states, after which the next stage is triggered.

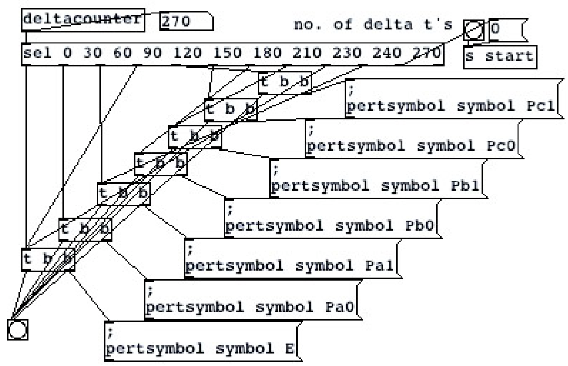

Figure 10: The Protocol of *Analogique B* in Pd

The Mechanisms

Each state is treated as an individual mechanism, behaving according to the probabilities determined by the protocol and current stage. A uniform random number is generated from 0-999, which chooses the next screen based on the current screen and the MTPZ.

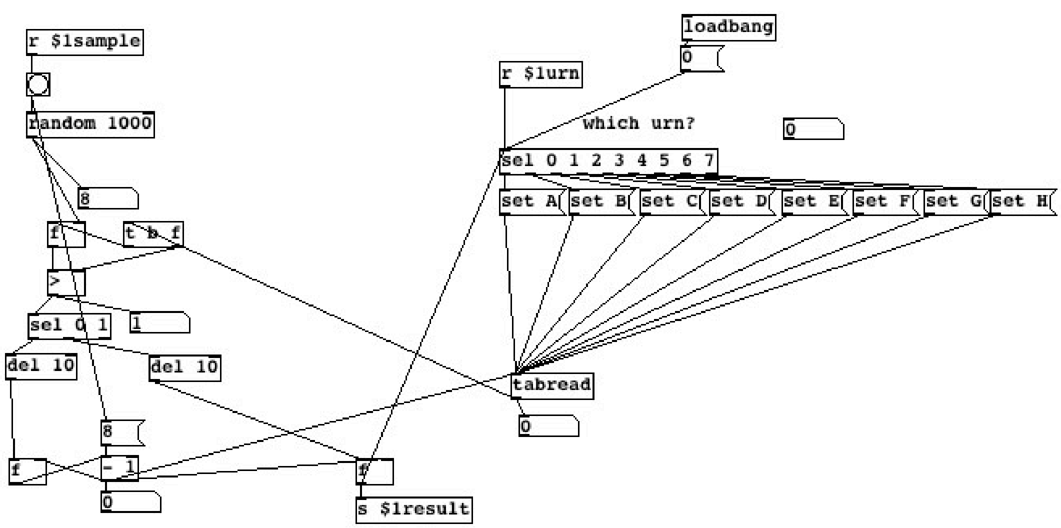

Figure 11: An Individual Mechanism from *Analogique B* in Pd

States (Internal Mechanisms)

All thirty states are determined at the beginning of each stage. Every 0.5 seconds, the patch generates grains according to the pre-determined states until all thirty have been used. Then the next stage is triggered the the thirty states receive their new values from their internal mechanisms.

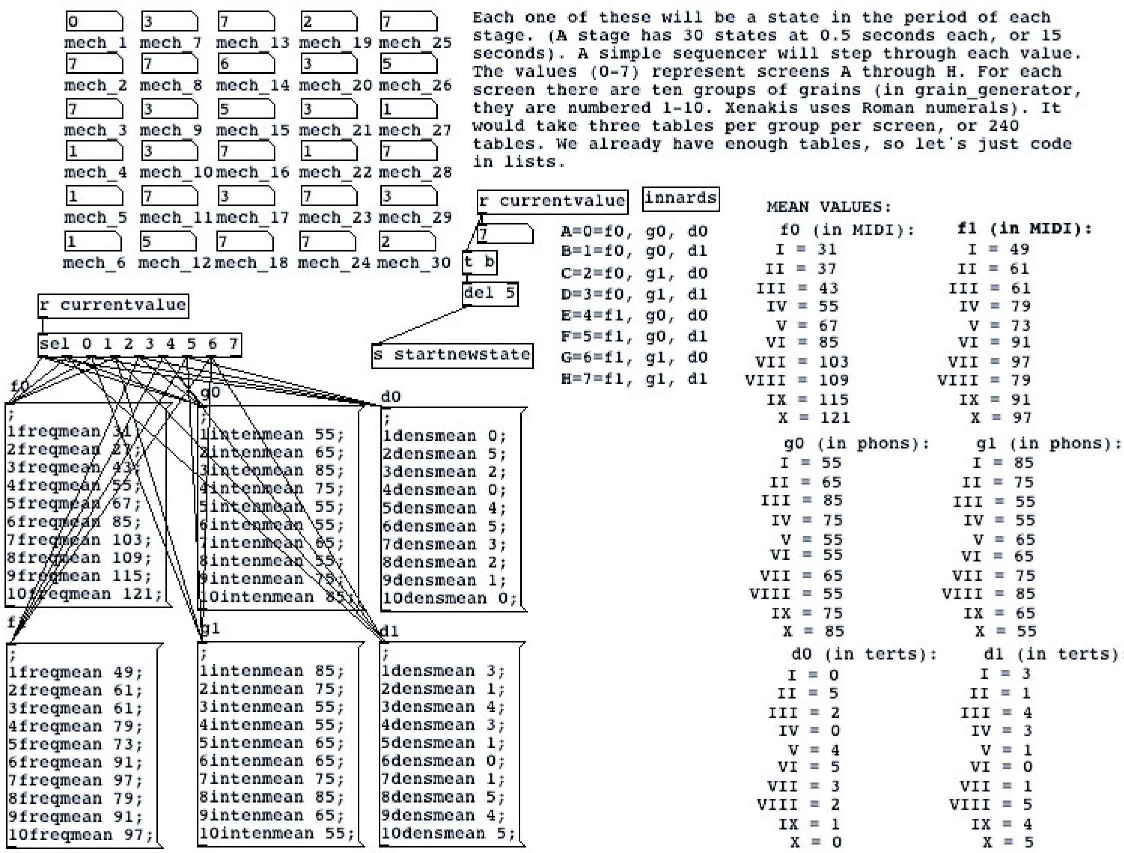

Figure 12: States and Internal Mechanisms in Pd

Frequency, Intensity, Density, and Temporal Distributions

Each state is a numeric representation of the eight screens (0=A, 1=B, 2=C, etc.). Each screen indicates ten sets mean values for each of frequency, intensity and density. A grain generator receives one set and begins generating frequency and intensity values according to a Gaussian distribution. The grain generator also determines the mean time between grains by using the mean density (grain per time) and the duration of a state. A Poisson random number generator sends values of time between grains. The grain generator tends to create a grain at the beginning of each stage, the culmination of which over time creates a steady 120m.m. beat. Xenakis never intended for a cloud to be so closely restrained within its own stage, that grains are artificially flattened on the screen (Xenakis 1992: 50-51). This required foreknowledge of what was to occur, an unavailable commodity in real time.

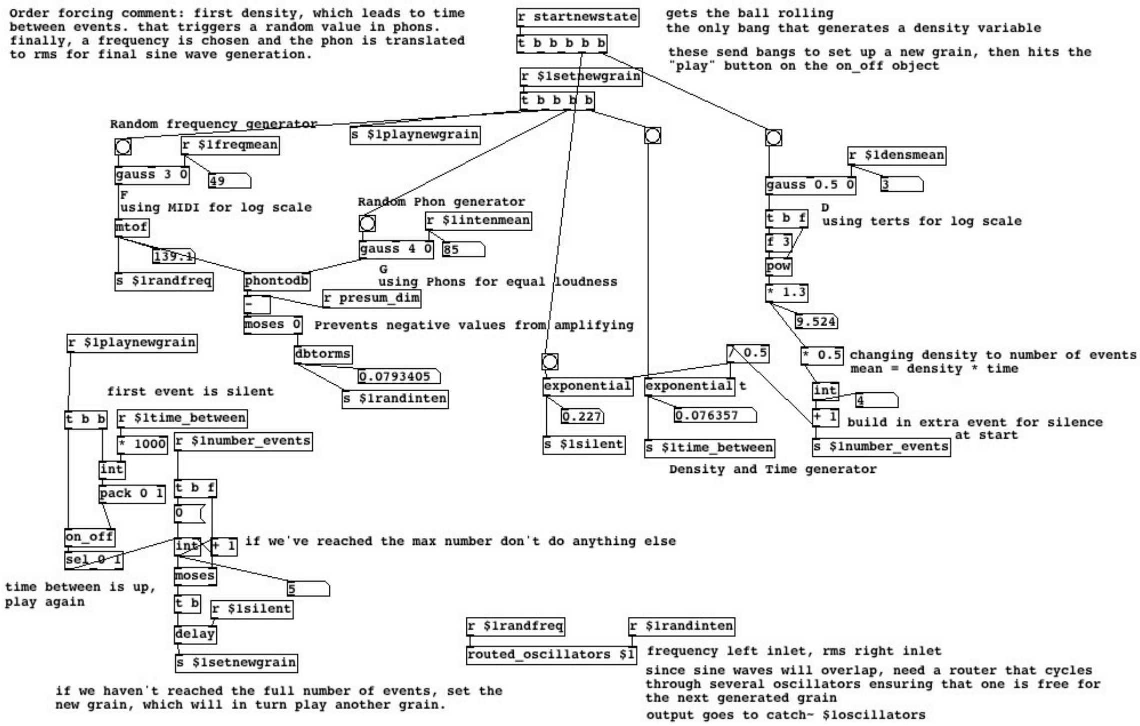

Figure 13: A Single Grain Generator in Pd

Concessions to Psychophysiology

Xenakis constructed his screens so they were linear in perception, not in its physical parameters. Though his grids are square, he assumed a mapping onto a topography determined by the psychophysiological response to equal-loudness curves and a logarithmic scale for density. To ensure that the grain generators create clouds on square perceptual grids, the patch uses MIDI values to generate the frequencies and a special “phontodB” look-up table to generate the intensities. This maintains the mapping indicated by Xenakis by his figure (Xenakis 1992: 48):

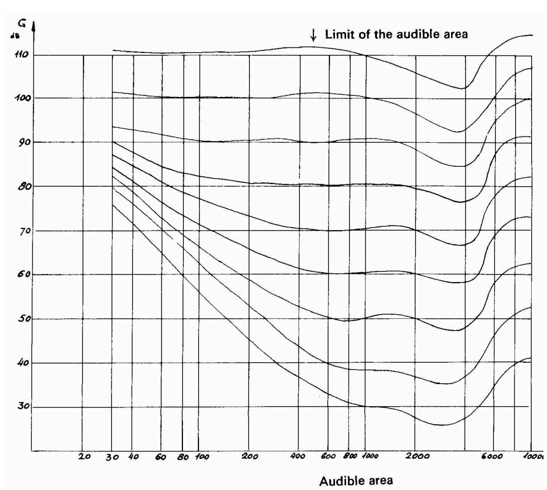

Figure 14: Equal-loudness curves

Sonic Result – Conclusion

There are a number of significant differences between Xenakis' original and the re-creation. The first obvious difference is the quality of sound. The analog tape sound is more complex; it appears that Xenakis used some kind of reverberation to enrich the sound of the tape. Furthermore, clipping or popping will surface in the computer version if the right combinations of grains appear, a situation easily avoided when a composer is making the choices

Other differences exist beyond the obvious quality differences. These differences lead to more profound conclusions. One of the most visceral differences emerges from the fact that each state is so clearly heard. Namely, the perturbations sound distinctly more homogeneous than the equilibrium states. The second-stage perturbations have a homogeneous quality with a bit more variety. In effect, the stages themselves are perceptually more acute, more tangible, and more distinctive. The "protocol of strains and relaxations" are more present between the stages than in the original. Secondly, the grains merge more completely to form a complex sound, while in Xenakis' version some of the higher frequencies poke through the texture to sound like a squealing musical line

Yet, there is also something distinctly lacking in the real-time computer version, something human missing from the process, something sensed if not easily identified. It is a difficult quality to express or quantify. In a phrase, there seems to be a peculiar absence of intention. In essence, the *process* of the work is more clearly heard in the real-time version; but the *art* of the work exists in Xenakis' carefully handcrafted tape version.

I believe the reason lies in the construction of the details. Imagine the construction of *Analogique B*: Xenakis approximated, by hand, the results of certain random variables. These variables determined the number of grains, the frequencies of a cloud, even the intensity. Yet, Xenakis had to sit in front of a tape machine and record these grains by hand. He had to listen for intensity, making adjustments as necessary. And, when he placed the grains in time, though he might have had calculations to determine where in time he placed a grain, it did not specify which grain.

Metaphorically, Xenakis had a basket of tape snippets for each cloud of grains. They contained proportions of frequencies and intensities as required by their random distribution. Also, he had a kind of "map" of points in time these tape snippets may be placed. Then, he *chose* which tape snippets to place on the map of points. The answer seems to be, then, that the re-creation lacks *the composer's will*, a philosophical point that Xenakis often made when comparing his stochastic music to certain determinate and in-determinate works of his contemporaries (Xenakis 1992: 77, 181).

Real-time stochastic methods allow a composer to sculpt a work at a macro- level of composition. But, *Analogique B* was designed to have the macro-composition set beforehand, while the ability to sculpt the micro-compositional elements remained. By using real-time methods in the manner in which *Analogique B* was created, there leaves no opportunity for human intervention. It stands to reason, then, that had Xenakis had access to real-time methods, he might have approached the construction of the piece differently than he did.

Cited Source

Xenakis, Iannis. 1992. *Formalized Music: Thought and Mathematics in Composition*, rev ed. Stuyvesant, NY: Pendragon Press.